▪ First Order Differential Equations

(1)Solve the following differential equations:

$(i)\ x\,y` = \cot y$ $(ii)\ \tan y\,dx + \tan x\,dy = 0$

$(iii)\ (y-4)dx - yx^2dy = 0$ $(iv)\ (x+1)e^{-y}dx + xydy = 0$

$(v)\ y` = (y^2+1)x^2$ $(vi)\ yy` = (y^2+1)\cos x$

(2)Solve the following differential equations:

$(i)\ (y+x)dx - (x-y)dy = 0$ $(ii)\ y` = \frac{y}{x} + \cot\frac{y}{x}$

$(iii)\ y` = \frac{y}{x} + \sinh\frac{y}{x}$ $(iv)\ y` = \frac{y}{x} + \cosh\frac{y}{x}$

$(v)\ (x-y)dx + (x-y+1)dy = 0$ $(vi)\ (x-y+2)dx + (3x-y)dy = 0$

(3)Solve the following differential equations:

$(i)\ (2x+6xy^2)dx + (3y^2+6x^2y)dy = 0$ $(ii)\ (x^2+y\cos x)dx + (y+\sin x)dy = 0$

$(iii)\ (4x^3+2xe^y)dx + (8+x^2e^y)dy = 0$ $(iv)\ (x-\sin y)dy + (y+\ln x)dx = 0$

$(v)\ (y+\tan y)dx + x(1+sec^2y)dy = 0$ $(vi)\ (y+x\sinh y)dy + (2+\cosh y)dx = 0$

(4)Solve the following linear differential equations:

$(i)\ y` + \frac{1}{x}y = 8x^2$ $(ii)\ y` + y\cot x = \csc x$

$(iii)\ y` - \frac{2}{x}y = 4$ $(iv)\ y` + \frac{2}{x+2}y = x$

$(v)\ y` + y\,tan\,x = sec\,x$ $(vi)\ y` + 2y = x + 2\,cosh\,x$

(5)Solve the following differential equations:

$(i)\ y\ln x\,dx + (1+4y^2)dy = 0$ y(1) = 1

$(ii)\ (y-x+1)dx + (y-x-1)dy = 0$ y(0) = 3

$(iii)\ (4x+ye^{xy})dx + (2y+xe^{xy})dy = 0$ y(0) = 2

$(iv)\ y` + \frac{1}{x}y = 4\ln x$ y(1) = 2

$(v)\ y` - y = 4x\,e^x$ y(0) = 1

(6)Find the orthogonal trajectories of the following family of curves:

(i) $(x-c)^2 + y^2 = c^2$ (ii) $x^2 + y^2 - cy = 0$

(iii) $x^2 - y^2 = c$ (iv) $x^2 + 2y^2 = c$

(v) $2x - 5y = c$ (vi) $x^2 + y^2 - cx = 0$

▪ Higher Order Differential Equations

(1)Solve the following differential equations:

$(i)\ y`` - 7y` + 6y = 0$ $(ii)\ (D^2 - 6D + 5)y = 0$

$(iii)\ y`` + 8y` + 16y = 0$ $(iv)\ (D^2 - 6D + 9)y = 0$

$(v)\ y`` + 25y = 0$ $(vi)\ (D^3 - 6D^2 + +12D - 8)y = 0$

$(vii)\ y``` - 4y`` - 5y` = 0$ $(viii)\ (D^4 - 16)y = 0$

(2)Solve the following differential equations:

$(i)\ y`` + 4y` + 3y = 5e^{2x} + 12e^{3x}$ $(ii)\ (D^2 - 3D + 2)y = 6 + 4e^{3x}$

$(iii)\ y`` - 3y` + 2y = (3 - e^{-2x})^2$ $(iv)\ (D^2 - 4D + 4)y = 2\cosh 2x$

$(v)\ y`` - 5y` + 6y = 4e^{3x} + e^{2x}$ $(vi)\ (D^3 - 3D^2 + 3D - 1)y = 2 + e^x$

(3)Find the solution of the following equations:

$(i)\ y`` + 2y` + y = \sin 2x$ $(ii)\ (D^2 - 2D - 3)y = \cos 2x$

$(iii)\ y`` + 9y = e^{3x} + \cos 3x$ $(iv)\ (D^2 - 4)y = 2cos^2 3x$

$(v)\ y`` + y = (\sin x + \cos x)^2$ $(vi)\ (D^3 + 2D^2 - 3D)y = \cos x$

(4)Find the solution of the following equations:

$(i)\ y`` + y = x^3$ $(ii)\ (D^2 + D + 2)y = 3 + x^2$

$(iii)\ y`` + 3y` + 2y = x + x^2$ $(iv)\ (D - 2)^2 y = x^2 e^{2x}$

$(v)\ y`` - 2y` + y = e^x \cos 2x$ $(vi)\ (D^2 - 5D + 6)y = e^{3x} \sin x$

$(vii)\ y`` + 3y = 2^x$ $(viii)\ (D^2 + D)y = \sin 3x \,.\cos x$

(5)Find the solution of the following equations:

$(i)\ y`` + 9y = \sec 3x$ $(ii)\ (D^2 + 1)y = \ln\cos x$

$(iii)\ y`` + 16y = \tan 4x$ $(iv)\ (D^2 + 4)y = \csc 2x$

$(v)\ y`` - 4y = \dfrac{1}{1 + e^{2x}}$ $(vi)\ (D^2 - 4D + 4)y = \dfrac{e^{2x}}{x}$

(6)Solve the following initial value problems:

$(i)\ y`` - y` - 2y = 4e^{-x} + 6e^{2x}$ $y(0) = 0,\quad y`(0) = 1$

$(ii)\ y`` + y` + y = 1 + x^2$ $y(0) = y`(0) = 2$

▪ Laplace Transformations

I-Find F(s) of each of the following functions:

(1) $f(t) = t^3 + 3t^2 - 2$

(2) $f(t) = \sqrt{t} + (2+t)t^2$

(3) $f(t) = t^2(2+t)^2$

(4) $f(t) = e^{3t} - t^2$

(5) $f(t) = \sin 2t + 3t^2 + \cosh t$

(6) $f(t) = e^{2t} + \cos 3t - 2\sinh t$

(7) $f(t) = sin^2 2t + cos^2 t$

(8) $f(t) = 3 + cos^3 2t$

(9) $f(t) = \sin 3t \,.\cos t$

(10) $f(t) = \sin 3t \,.\sin 2t$

(11) $f(t) = t^2 .\sin 2t$

(12) $f(t) = 2 + e^{3t}\cos 4t$

(13) $f(t) = (t + \sin t)(3 - \cos 2t)$

(14) $f(t) = e^{-t}\sinh 3t$

(15) $f(t) = \sin 2t \,.\cosh 3t$

(16) $f(t) = \cos t \,.\sinh 2t$

(17) $f(t) = t + sinh^2 t$

(18) $f(t) = cosh^2 t.\sin 3t$

(19) $f(t) = t.e^{3t}.\cos 2t$

(20) $f(t) = e^{-t}.t.\sin 2t$

(21) $f(t) = 3^t.\cos 2t$

(22) $f(t) = 2^t.t - \sin 2t$

(23) $f(t) = (t-2)^4, \;\; t > 2$

(24) $f(t) = \sin(t - \frac{\pi}{4}), \quad t > \frac{\pi}{4}$

(25) $f(t) = 2 + t, \;\; t \geq 3$

(26) $f(t) = \sin(t - \frac{\pi}{4})$

(27) $f(t) = \begin{cases} 2, & 0 \leq t \leq 1 \\ t, & t > 1 \end{cases}$

(28) $f(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ t^2, & t > 1 \end{cases}$

(29) $f(t) = \dfrac{\sin 2t}{t}$

(30) $f(t) = \dfrac{\cos 2t - \cos t}{t}$

(31) $f(t) = \dfrac{e^{-t} - e^{-2t}}{t}$

(32) $f(t) = \dfrac{\cos t - \cos 3t}{t}$

(33) $f(t) = \partial_0 + t$

(34) $f(t) = \partial_3 . t$

(35) $f(t) = \partial_2 . e^{3t}$

(36) $f(t) = U(t-3)$

II- Find the inverse Laplace transform of each of the following:

(1) $F(s) = 2 + \frac{3}{s}$ (2) $F(s) = \frac{3}{s-2} + \frac{1}{s^3}$ (3) $F(s) = \frac{1}{s^2-4} + \frac{3}{s^2+4}$

(4) $F(s) = \frac{s+1}{s^2+1}$ (5) $F(s) = \frac{1}{s+2} + \frac{1}{(s-2)^3}$ (6) $F(s) = \frac{s-4}{s^2-4} + \frac{3s}{s^2+4}$

(7) $F(s) = \frac{s}{s^2-3s+2}$ (8) $F(s) = \frac{s+4}{s^3-4s^2+3s}$ (9) $F(s) = \frac{3}{s^2-4s+4}$

(10) $F(s) = \frac{s}{s^2-2s+2}$ (11) $F(s) = \frac{1}{s(s-2)^3}$ (12) $F(s) = \frac{3}{s^2-4s-5}$

(13) $F(s) = \frac{2}{s(s^2+1)}$ (14) $F(s) = \frac{1}{s^2(s^2+1)}$ (15) $F(s) = \ln\frac{s+1}{s+2}$

(16) $F(s) = \tan^{-1} s$ (17) $F(s) = \tan^{-1}(1+s)$ (18) $F(s) = e^{-2s}$

(20) $F(s) = \frac{1}{s^2}e^{-3s}$ (20) $F(s) = \frac{s^2+1}{s^2+4}$ (21) $F(s) = \frac{s^2-4}{s^2-1}$

III- Solve the following differential equations:

(1) $y` - 2y = 0,$ $y(0) = 2$ $y = 2e^{2t}$

(2) $y`` + 4y` - 5y = 0,$ $y(0) = 1,\ y`(0) = 0$ $y = (\cosh 3t + \frac{2}{3}\sinh 3t)e^{-2t}$

(3) $y`` + 5y` + 6y = 0,$ $y(0) = y`(0) = 1$ $y = 4e^{-2t} - 3e^{-3t}$

(4) $y`` + 2y` + 5y = 0,$ $y(0) = y`(0) = 1$ $y = (\sin 2t + \cos 2t)e^{-t}$

(5) $y` + 3y = 1 + t,$ $y(0) = 0$ $y = \frac{2}{9} + \frac{1}{3}t - \frac{2}{9}e^{-3t}$

(6) $y`` + 4y` + 3y = e^{-t},$ $y(0) = y`(0) = 1$ $y = \frac{1}{2}te^{-t} + \frac{7}{4}e^{-t} - \frac{3}{4}e^{-3t}$

(7) $y`` - 3y` + 2y = 6e^{2t},$ $y(0) = y`(0) = 3$ $y = 9e^{t} - 6e^{2t} + 6te^{2t}$

▪ Numerical Analysis

Solution of Equation of One Variable

(1)Find a root for each equation using the bisection method:

(i) $f(x) = x^3 - 5x - 7 = 0$ in the interval [2, 3]

(ii) $f(x) = x^5 - 2x - 5 = 0$ in the interval [1, 2]

(iii) $f(x) = x^4 + 4x - 10 = 0$ in the interval [1, 2]

(iv) $f(x) = e^x + 4x - 2 = 0$ in the interval [0, 1]

(v) $f(x) = 5x^3 + x^2 - 2 = 0$ in the interval [0, 1]

(vi) $f(x) = x^4 + 2x^2 - x - 3 = 0$ in the interval [1, 2]

(vii) $f(x) = x - e^{-x} = 0$ in the interval [0.1, 1]

(2)Find a root for each equation using the Newton's method:

(i) $f(x) = x^3 - 3x^2 + 1 = 0$ in the interval [0, 1]

(ii) $f(x) = x^4 + x^3 + x - 2 = 0$ in the interval [0, 1]

(iii) $f(x) = x + e^x = 0$ in the interval [–1, – 0.1]

Curve Fitting

(1)Find the line that fits the following data:

(i) (2, 3), (4, 4), (6, 7), (8, 9), (10, 12)

(ii) (1, 2), (2, 3), (3, 5), (4, 8), (5, 9)

(iii) (1, 2), (1.5, 3), (2, 4.4), (2.5, 6), (3, 8)

(iv) (0.2, 1.4), (0.4, 2), (0.6, 2.5), (0.8, 3), (1, 4)

(v) (10, 1), (20, 3), (30, 4), (40, 5), (50, 8)

(2)Find the parabola that fits the above data.

(3)Find the logarithmic curve that fits the above data.

(4)Find the exponential curve that fits the above data.

Interpolation

(1)Find the table of finite differences of the data:

(1, 1), (2, 7), (3, 25), (4, 61), (5, 121) and find y at x = 0, 1.5

(2)Find the table of divided differences of the data:

(1, 6), (2, 11), (4, 27), (7, 66), (12, 171) and find y when x = 0, 3

(3)Compute the polynomial that satisfies the data:

(1, 2), (2, 5), (4, 6), (5, 10) and find y when x = 0, 3

(4)Evaluate the general term of the sequence: 3, 13, 35, 75,…

(5)Find the Lagrange's polynomial $P_2(x)$ to find $P_2(1)$, $P_2(5)$ from the data:

(0, 2), (2, 3), (3, 12), (4, 17), (6, 20), (7, 24).

(6)Find the Lagrange's polynomial $P_3(x)$ to find $P_2(6)$ from the data:

(1, 3), (3, 7), (4, 10), (5, 12), (7, 18), (8, 24).

(7)Compute the value of x at y = 4 from the data: (1, 3), (2, 5), (3, 9), (4, 13)

(8)Find a root to each of the following equations:

(i) $x^4 + x^2 - 3 = 0$ in the interval [1, 2]

(ii) $x^5 + x^2 - 1 = 0$ in the interval [0, 1]

(iii) $e^{-x} + 3x - 2 = 0$ in the interval [0, 1]

Numerical Differentiation and Integration

(1)Find y` at x = 2 from the data: (1, 1.73), (1.5, 1.69), (2, 1.51), (2.5, 1.42).

(2)Find y` at x = 1, x = 4 from the data: (1, 3), (2, 6), (4, 5), (6, 8).

(3)Find y`(1) from the following data : (1, 3), (2, 12), (4, 48), (7, 327).

(4)Find y`(0), y`(1) and y`(5) from the data: (0, 1), (1, 4), (2, 15), (3, 40), (5, 60).

(5)Find $f`(0)$ $where$ $f(x) = \begin{cases} 2^x, & x > 0 \\ x^2 + 1, & x \le 0 \end{cases}$

(6)Find $f`(2)$ $where$ $f(x) = |x - 2|$.

(7)Find $f`(3)$ $where$ $f(x) = (x-3)^{\frac{2}{3}}$.

(8)Find the following integrals using trapezoidal rule:

(i) $\int_0^2 \frac{1}{\sqrt{1+x}} dx$ (ii) $\int_1^2 \frac{1}{x} 3^x dx$ (iii) $\int_1^\infty \frac{1}{1+x^2} dx$ (iv) $\int_1^\infty \frac{x}{1+x^3} dx$ (v) $\int_1^\infty \frac{1}{1+x^4} dx$

(9)Evaluate the following integrals using Simpson's rule:

(i) $\int_0^2 \frac{1}{\sqrt{1+x}} dx$ (ii) $\int_1^2 \frac{1}{x} 3^x dx$ (iii) $\int_2^3 \frac{x}{\ln x} dx$ (iv) $\int_0^2 \frac{x}{\sqrt{1+x}} dx$ (v) $\int_1^2 \sqrt{1+\ln x}\, dx$

Solution of Differential Equations

(1)Solve the following equations by Taylor's method:

(i)$y' = x y^2 + 2$, $y(0) = 1$ (ii) $y' = x^2 + y^2$, $y(0) = 2$

(iii)$y' = y \cos x$, $y(0) = 1$ (iv)$y'' = y x^2$, $y(0) = y'(0) = 3$

(2)Solve the following equations by Picard's method:

(i)$y' = y e^x$, $y(0) = 1$ (ii) $y' = y + x^2$, $y(0) = 2$

(iii)$y' = y + \sin x$, $y(0) = 1$ (iv)$y' = x + y$, $y(0) = 0$

(3)Solve the following equations by Euler's method:

(i)$y' = \sqrt{x + y}$, $y(0) = 1$ (ii) $y' = x + \sqrt{y}$, $y(0) = 2$

(iii) $y' = x + y$, $y(0) = 3$ (iv) $y' = xy$, $y(0) = 4$

where x in [0, 0.4]

(4)Solve the following equations by Runge-Kutta method, where x in [0, 0.3] :

(i)$y' = \sqrt{x + y}$, $y(0) = 1$ (ii) $y' = x + \sqrt{y}$, $y(0) = 2$

(iii) $y' = \ln(x + y)$, $y(0) = 2$ (iv) $y' = x y$, $y(0) = 4$